KL (DEEMED TO BE UNIVERSITY)

CDOE

CENTER FOR DISTANCE AND ONLINE EDUCATION

BUSINESS MATHEMATICS

BBA

How to Use Self-Learning Material?

The pedagogy used to design this course is to enable the student to assimilate the concepts with ease. The course is divided into modules. Each module is categorically divided into units or chapters. Each unit has the following elements:

- **Table of Contents:** Each unit has a well-defined table of contents. *For example: "1.1.1. (a)" should be read as "Module 1. Unit 1. Topic 1. (Sub-topic a)" and 1.2.3. (iii) should be read as "Module 1. Unit 2. Topic 3. (Sub-topic iii).*
- **Aim:** It refers to the overall goal that can be achieved by going through the unit.
- **Instructional Objectives:** These are behavioural objectives that describe intended learning and define what the unit intends to deliver.
- **Learning Outcomes:** These are demonstrations of the learner's skills and experience sequences in learning, and refer to what you will be able to accomplish after going through the unit.
- **Self-Assessment Questions:** These include a set of multiple-choice questions to be answered at the end of each topic.
- **Did You Know?:** You will learn some interesting facts about a topic that will help you improve your knowledge. A unit can also contain Quiz, Case Study, Critical Learning Exercises, etc., as metacognitive scaffold for learning.
- **Summary:** This includes brief statements or restatements of the main points of unit and summing up of the knowledge chunks in the unit.
- **Activity:** It actively involves you through various assignments related to direct application of the knowledge gained from the unit. Activities can be both online and offline.
- **Bibliography:** This is a list of books and articles written by a particular author on a particular subject referring to the unit's content.
- **e-References:** This is a list of online resources, including academic e-Books and journal articles that provide reliable and accurate information on any topic.
- **Video Links:** It has links to online videos that help you understand concepts from a variety of online resources.

Author Profile

Mr. Alajangi Suresh, M.Sc., MPhil., is presently working as Content Writer in K L (Deemed to be) University. He received his M.Sc. in Mathematics from Osmania University and his M.Phil. in Mathematics from Sri Venkateshwara University. The area of his research is on Fixed Point theory. He submitted Fixed Point theorems in complete metric space using Ćirić's operators, commutative, asymptotic commutative conditions and applications for his M.Phil. degree in 2011.

He lectured for about three years in junior and degree colleges. He worked as assistant professor in Aditya Engineering College, Surampalem, East Godavari District, Andhra Pradesh, for about three years. He worked as a Content Developer (Subject Matter Expert) for about 3 years in chegg.com from 2012 to 2015. He also worked as a freelance Content Developer in chegg.com for about six years. He has about 14 years of experience in teaching and in Industrial discipline.

Business Mathematics

Course Description

Business Mathematics is about mathematical concepts which are used in business area. The challenges in business need to be dealt with in a proper approach and effectively solved. Business Mathematics not only uses calculations and estimations but also analyses business problems and works upon them. When we describe mainly about estimations, profit, loss and interest, tax calculations and salary calculations the role of Business Mathematics is phenomenal. Business Mathematics helps to finish the business tasks efficiently. Business Mathematics deals in finding profit, margin, cash discount, trade discount of a product using the cost of a product. The commercial companies use Business Mathematics to manage and record business works. The corporate companies use Business Mathematics in accounting, inventory management, sales forecasting, marketing, and financial analysis. The financial activities are controlled by Business Mathematics. Business Mathematics helps in projections of the revenue and expenses of a business. If one needs to analyse the financial position of a business, Business Mathematics role becomes crucial.

Business Mathematics takes an important role in the analysis on increase or decrease in sales figures or pricing figures. Business Mathematics is used to identify the contributions of each employee to the business to monitor employees performance at every level help take the business to a higher level used in maintaining the smooth operation of a company to assess the financial performance of the business to estimate the incomes and expenditures and the risk analysis to assess the business competitors, their strong areas and their business strategies to take and to improve decision making about different aspects of business such as costs, raw materials, marketing, and advertising, strategies for the short and long term, etc. Business Mathematics helps to analyse unforeseen situations and to take necessary steps in favour of the company. Business Mathematics is used to understand the situations using graphs and figures and it helps in nullifying or minimising the damages happening to the company. Business Mathematics gives a detailed case study occurring in different business situations. Learning and utilising Business Mathematics enhances our abilities, sharpens our thinking, and helps in precisely formulating and estimations. The concepts of Business Mathematics help in illustrating and analysing many real-life situations. Business Mathematics provides the mathematical models that one needs to study in order to manage the records of all the essentials involved in a business.

The **Business Mathematics** Course contains F**our Modules.**

MODULE 1: ELEMENTS OF MATRIX ALGEBRA

Introduction - Types of Matrices - Scalar Multiplication of Matrix Equality of Matrices Matrix operations - Transpose of Matrix - Determinants of Square Matrix - Inverse of Matrix - Solutions of Simultaneous equations with the inverse of a Matrix - Rank of a Matrix – Eigen values.

MODULE 2: INTRODUCTION TO VARIABLES AND FUNCTIONS

Meaning of a variable, Types of variables - Dependent variable and Independent variable - Categorical, Discrete and Continuous variables - Mediating and Moderating variables - Variables vs. Attributes. Basic Concept of Functions - Types of Functions - Linear Function - Constant Function - Quadratic Functions - Exponential Functions - Homogeneous Functions – Logarithm functions.

MODULE 3: INTRODUCTION TO CALCULUS

Calculus definition - Types of calculus - Limits - Differentiation - Derivatives of Functions - Rules of Derivatives - Second order Derivatives - Partial derivatives - Application of Derivatives - Integration - Definite integrals - Indefinite integrals - Application of integration.

MODULE 4: INTRODUCTION TO FINANCIAL MATHEMATICS

Progressions - Arithmetic Progressions - Geometric - Simple Interest - Compound Interest - Problems with Business applications

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MODULE 2

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Unit 2.1 - Introduction to Variables
Unit 2.2 - Introduction to Functions

MODULE 3

Introduction to Calculus

Unit 3.1 - Limits and Differentiation
Unit 3.2 - Applications of Derivatives
Unit 3.2 - Integration and Applications of Integration

MODULE 4

Introduction to Financial Mathematics

Unit 4.1 - Progressions, Arithmetic Progressions and Geometric Progressions
Unit 4.2 - Simple Interest and Compound Interest
Unit 4.2 - Problems with Business Applications

Business Mathematics

MODULE1

Elements of Matrix Algebra

MODULE - 1

Elements of Matrix Algebra

Module Description

The usage of matrices is necessary in various stages of business such as sales figures projection, cost estimation, price estimation, revenue calculations, demand and supply analysis. Matrices help in analysing the results of the business marketing strategies. Matrices are a powerful tool in mathematics. Matrices are useful in writing and solving simultaneous equations. Matrix multiplication is useful feature to get linear transformations. Matrices simplify our work to a great extent.

In different business purposes, the business industry uses the computers to store their data in the form of electronic spreadsheet programs. These are prepared using Matrices. The use of matrices is vital in business, economics and industrial management.

The module has two chapters.

Unit 1.1
Introduction to Matrices

Unit 1.2
Determinants, Inverse of Matrix and Simultaneous Equations

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Aim

To define matrix and to describe the types of matrices, matrix operations and transpose of matrices.

Instructional Objectives

After completing this unit, you will be able to:

- Define the concept of Matrix,
- Identify the types of matrices
- Illustrate matrix operations and transpose of matrices

Learning Outcomes

At the end of this unit, you are expected to:

- Recall the definition of matrix
- Use the types of matrices
- Describe the matrix operations and transpose of a matrix.

1.1.1 Introduction

Economists and Industrial Management widely use matrices for accounting and for their input and output tables of data. Matrices are very helpful in linear programming. Matrices are used as data representation and do the data manipulations easily according to the user needs. The linear programming techniques that are based on data of matrices are useful to maximise profit. Matrices are used to present the production details and raw materials availability of an Industry. Matrices play a vital role in the projection of three-dimensional image into a two-dimensional screen, creating a realistic seeming motion in computer applications.

Self-Assessment Questions

1. Economists and Industrial Management widely use matrices for accounting and for their input and output tables of ____________

a) Account

b) Content

c) Manipulations

d) Data

2. ________are used as data representation and do the data manipulations easily according to the user needs.

a) Table

b) Matrices

c) Numbers

d) Diagrams

3. What techniques that is based on data of matrices are useful to maximise profit.

a) Matrices

b) Accounting

c) Linear Programming

d) Data Manipulations

1.1.2 Definition of Matrix

A matrix is an ordered rectangular arrangement or array of numbers or functions. Here the numbers or functions are called the elements or the entries of the matrix. If a matrix has m rows and n columns, then the order of the matrix is, $m \times n$

A matrix with m rows and n columns is denoted by , $\left[a_{ij}\right]_{m\times n}$

The element a_{ij} represents the entry of ith row and jth column of the matrix.
Matrices are usually denoted by capital letters of the alphabets A, B, C,…

For example,

$$A=\begin{bmatrix} 2 & 3 & 0 \\ 5 & 26 & 1 \end{bmatrix}, B=\begin{bmatrix} x^2+2 & 5x-3 & 4x^3-2x+3 \\ 2x+11 & x+2 & x-10 \\ x^2+x+1 & 0 & 15x+3 \end{bmatrix}$$

In the above A is a 2 x 3 order matrix because A has 2 rows and 3 columns.

Similarly B is 3 x 3 order matrix.

In matrix A,

$$a_{11}=2, a_{12}=3, a_{13}=0$$
$$a_{21}=5, a_{22}=26, a_{23}=1$$

In matrix B,

$$B=\begin{bmatrix} x^2+2 & 5x-3 & 4x^3-2x+3 \\ 2x+11 & x+2 & x-10 \\ x^2+x+1 & 0 & 15x+3 \end{bmatrix}$$
$$a_{11}=x^2+2, a_{12}=5x-3, a_{13}=4x^3-2x+3$$
$$a_{21}=2x+11, a_{22}=x+2, a_{23}=x-10$$
$$a_{31}=x^2+x+1, a_{32}=0, a_{33}=15x+3$$

Self-Assessment Questions

4. The entry of the matrix is either …….. or……..

a) Numbers, Functions

b) Entries, Names

c) Rows, Columns

d) Symbols, Units

5. Find $a_{11}, a_{13}, a_{22}, a_{32}$ from the following matrix.

$$A = \begin{bmatrix} 3 & -2 & 6 \\ 5 & 8 & 4 \\ 7 & 2 & 1 \end{bmatrix}$$

a) 5,6,2,1

b) 3,6,8,2

c) 6,3,2,1

d) 8,6,2,3

6. Find the order of the following matrix $C = \begin{bmatrix} 6 & -1 & 2 \\ 5 & 6 & 4 \\ 9 & 3 & 3 \\ 5 & -3 & 2 \end{bmatrix}$

a) 3 x 2

b) 3 x 3

c) 1 x 3

d) 4 x 3

1.1.3 Types of Matrices

1.1.3.1 Column Matrix

The matrix which has only one column, then the matrix is called column matrix.

In general, the matrix is in the form, $A = [a_{ij}]_{m\times 1}$

Here $m\times 1$ is the order of the matrix.

For example, $A = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

Here A is a 3 x 1 matrix.

1.1.3.2 Row Matrix

The matrix which has only one row, then the matrix is called row matrix.
In general , the matrix is in the form
Here 1 x m is the order of the matrix. $A = [a_{ij}]_{1\times m}$
For example, $B = [5 \quad -2 \quad 4]$

Here B is 1 x 3 matrix.

1.1.3.3 Rectangular Matrix

In a matrix, if the number of rows is not equal to the number of columns, then the matrix is called Rectangular Matrix.
In general , the matrix is in the form $A = [a_{ij}]_{m\times n}$
Here $m \times n$ is the order of the matrix and

For example, $B = \begin{bmatrix} 2 & 3 & -5 \\ 6 & 7 & 8 \end{bmatrix}$

Here B is 2 x 3 matrix.

1.1.3.4 Square matrix

In a matrix, if the number of rows is equal to the number of columns, then the matrix is called Square Matrix.
In general, the matrix is in the form $A = [a_{ij}]_{m\times m}$
Here $m\times m$ is the order of the matrix.

For example, $C = \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}, D = \begin{bmatrix} 4 & 5 & 2 \\ 3 & 2 & 7 \\ 5 & 6 & 8 \end{bmatrix}$

Here C is 2 x 2 matrix and D is 3 x 3 matrix.

1.1.3.5 Diagonal matrix

A square matrix $E = \left[a_{ij}\right]_{m\times m}$ is said to be a diagonal matrix if all its non diagonal elements are zero.
A matrix $E = \left[a_{ij}\right]_{m\times m}$ is said to be a diagonal matrix if

The diagonal elements are $a_{11}, a_{22}, a_{33}, a_{44}, \ldots$

For Example, $E = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

1.1.3.6 Scalar matrix

A diagonal matrix $F = \left[a_{ij}\right]_{m\times m}$ is said to be a Scalar matrix if all its diagonal elements are equal.

A diagonal matrix $F = \left[a_{ij}\right]_{m\times m}$ is said to be a Scalar matrix if

For Example, $E = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

1.1.3.7 Identity matrix

In a square matrix, if the diagonal elements are 1 and the non diagonal elements are zero, then the matrix is called an identity matrix.

For example, $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

1.1.3.8 Zero matrix

In a matrix, if all the elements of the matrix are zero, then the matrix is said to be zero matrix or null matrix.

For example, $G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, H = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

1.1.3.9 Triangular Matrix

There are two types of Triangular matrices.

1. Lower Triangular Matrix

In a square matrix, if all of its elements above the main diagonal are zero then it is called Lower Triangular Matrix.

For example, $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 4 & 2 & 5 & 0 \\ 7 & 6 & 8 & 4 \end{bmatrix}$

2. Upper Triangular Matrix

In a square matrix, if all of its elements below the main diagonal are zero then it is called Upper Triangular Matrix.

For example, $B = \begin{bmatrix} 1 & 2 & 7 & 5 \\ 0 & 1 & 5 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

Self-Assessment Questions:

7.Find the number of columns in a column matrix and number of rows in a row matrix.

a) 1,3
b) 2,3
c) 1,1
d) 1,2

8.In which matrix the number of columns are equal to number of rows.

a) Square Matrix
b) Rectangular Matrix
c) Row Matrix
d) Column Matrix

9. Find the diagonal elements of the matrix $A = \begin{bmatrix} 3 & -2 & 6 \\ 5 & 8 & 4 \\ 7 & 2 & 1 \end{bmatrix}$

a) 6,8,7
b) 3,8,1
c) -2,8,1
d) -2,8,2

10. Find $4I$, I is a 3x3 unit matrix.

(a) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

11. Find Lower Triangular Matrix for the following matrix.

$$D = \begin{bmatrix} 5 & -2 & 6 & 3 \\ 8 & 2 & 4 & 5 \\ 7 & 2 & 5 & 3 \\ 2 & 9 & -2 & 4 \end{bmatrix}$$

(a) $\begin{bmatrix} 5 & 0 & 0 & 3 \\ 8 & 2 & 0 & 5 \\ 7 & 2 & 5 & 0 \\ 2 & 9 & -2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 0 & 6 & 3 \\ 8 & 2 & 0 & 5 \\ 7 & 2 & 5 & 0 \\ 2 & 9 & -2 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 8 & 2 & 0 & 0 \\ 7 & 2 & 5 & 0 \\ 2 & 9 & -2 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 8 & 2 & 0 & 5 \\ 7 & 2 & 5 & 0 \\ 2 & 9 & -2 & 4 \end{bmatrix}$

1.1.4 Scalar Multiplication of a Matrix

Let $A = \left[a_{ij}\right]_{m\times m}$ be a matrix. *k* be a scalar, *kA* be the matrix is obtained by multiplying each element of *A* by the scalar *k*

kA is the scalar multiplication of the matrix *A.*

The order of *kA* is equal to the order of *A.*

$$kA = k\left[a_{ij}\right]_{m\times m} = \left[k\left(a_{ij}\right)\right]_{m\times m}$$

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 8 & 9 & -3 \\ 6 & 7 & 9 \end{bmatrix},$$

$$2A = 2\begin{bmatrix} 2 & 3 & 5 \\ 8 & 9 & -3 \\ 6 & 7 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 6 & 10 \\ 16 & 18 & -6 \\ 12 & 14 & 18 \end{bmatrix}$$

Self-Assessment Questions:

12. Find the order of the matrix 3A when $A = \begin{bmatrix} 4 & 5 & 2 \\ 3 & 2 & 7 \\ 5 & 6 & 8 \end{bmatrix}$

(a) 3 x 1
(b) 3 x 3
(c) 1 x 3
(d) 4 x 3

13. Find 4B when $B = \begin{bmatrix} -2 & 6 & 3 \\ 5 & 2 & 7 \\ 9 & 6 & 1 \end{bmatrix}$

(a) $\begin{bmatrix} -8 & 6 & 12 \\ 5 & 8 & 7 \\ 36 & 24 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -2 & 24 & 12 \\ 20 & 8 & 28 \\ 9 & 6 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & 6 & 3 \\ 20 & 8 & 28 \\ 36 & 24 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} -8 & 24 & 12 \\ 20 & 8 & 28 \\ 36 & 24 & 4 \end{bmatrix}$

14. Find 4O where, $O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(a) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 4 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 0 & 0 \\ 4 & 0 & 0 \\ 4 & 0 & 0 \end{bmatrix}$

1.1.5 Equality of Matrices

Let $A = [a_{ij}], B = [b_{ij}]$ be two matrices with same order. Then A, B are said to be equal matrices if each element of A is equal to the corresponding element of B.

For example, $A = \begin{bmatrix} 4 & 5 & 2 \\ 3 & 2 & 7 \\ 5 & 6 & 8 \end{bmatrix}, B = \begin{bmatrix} 4 & 5 & 2 \\ 3 & 2 & 7 \\ 5 & 6 & 8 \end{bmatrix}, C = \begin{bmatrix} 4 & 3 \\ 2 & 3 \end{bmatrix}, D = \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$

Here A, B are equal matrices because the corresponding elements of A, B are equal.

C, D are not equal matrices because the corresponding elements of C, D are not equal.

Self-Assessment Questions

15. Suppose $A = \begin{bmatrix} 5 & 2 & 8 \\ 2 & 3 & 7 \\ 9 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and A,B are equal matrices.

Then find *a, b, c, d, e, f.*

(a) 5,7,3,2,8,2
(b) 5,3,8,2,2,7
(c) 5,8,3 2,2,7
(d) 5,2,8,2,3,7

16. Suppose $A=\begin{bmatrix}3 & 2\\ 5 & 1\end{bmatrix}, B=\begin{bmatrix}2x-1 & x\\ 2x+1 & x-1\end{bmatrix}$ and *A,B* are equal matrices. Then find the value of *x*.

(a) 3
(b) 2
(c) 5
(d) 1

17. Suppose $C=\begin{bmatrix}9 & 4\\ 5 & 1\end{bmatrix}, D=\begin{bmatrix}2x+3 & x+1\\ 2x-1 & x-2\end{bmatrix}$ and C, D are equal matrices. Then find the value of x.

(a) 2
(b) 3
(c) 9
(d) 1

1.1.6 Matrix Operations

1.1.6.1 Addition of matrices

Let $A=\left[a_{ij}\right], B=\left[b_{ij}\right]$ be two matrices with same order. Then the addition of *A, B* is *C = A+B* such that the corresponding elements of *A,B* are added. $C=\left[c_{ij}\right]=\left[a_{ij}+b_{ij}\right]$

For example, $A=\begin{bmatrix}6 & 7 & 1\\ 4 & 2 & 7\\ 5 & 6 & 8\end{bmatrix}, B=\begin{bmatrix}7 & 5 & 2\\ 1 & 2 & 8\\ 5 & 5 & 3\end{bmatrix}$

$$A+B=\begin{bmatrix}6+7 & 7+5 & 1+2\\ 4+1 & 2+2 & 7+8\\ 5+5 & 6+5 & 8+3\end{bmatrix}$$

$$=\begin{bmatrix}13 & 12 & 3\\ 5 & 4 & 15\\ 10 & 11 & 11\end{bmatrix}$$

Thus, $C=A+B$

$$=\begin{bmatrix}13 & 12 & 3\\ 5 & 4 & 15\\ 10 & 11 & 11\end{bmatrix}$$

1.1.6.2 Negation of a matrix

The negation of a matrix is obtained by replacing each element by its negation.

The negation of *A* is *-A*

$A = [a_{ij}]$ be a matrix.

The negation of $A = -A = [-(a_{ij})]$

1.1.6.3 Subtraction of matrices

$A = [a_{ij}], B = [b_{ij}]$ be two matrices with same order. Then the subtraction of *A, B* is *C = A-B* such that the corresponding elements of *A,B* are subtracted

That is, $C = [c_{ij}] = [a_{ij} - b_{ij}]$ such that $c_{ij} = a_{ij} - b_{ij}$

For example, $A = \begin{bmatrix} 6 & 7 & 1 \\ 4 & 2 & 7 \\ 5 & 6 & 8 \end{bmatrix}, B = \begin{bmatrix} 7 & 5 & 2 \\ 1 & 2 & 8 \\ 5 & 5 & 3 \end{bmatrix}$

$$A - B = \begin{bmatrix} 6-7 & 7-5 & 1-2 \\ 4-1 & 2-2 & 7-8 \\ 5-5 & 6-5 & 8-3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & -1 \\ 3 & 0 & -1 \\ 0 & 1 & 5 \end{bmatrix}$$

Thus, $C = A - B$

$$= \begin{bmatrix} -1 & 2 & -1 \\ 3 & 0 & -1 \\ 0 & 1 & 5 \end{bmatrix}$$

1.1.6.4 Properties of addition of matrices

1. Addition of matrices is Commutative.

That is $A + B = B + A$

For example, $A = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 \\ 6 & 8 \end{bmatrix}$

$$A + B = \begin{bmatrix} 1+3 & 3+2 \\ 5+6 & 2+8 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ 11 & 10 \end{bmatrix}$$

$$B + A = \begin{bmatrix} 3+1 & 2+3 \\ 6+5 & 8+2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 \\ 11 & 10 \end{bmatrix}$$

Thus, $A + B = B + A$

2. Addition of matrices is Associative

That is, $(A + B) + C = A + (B + C)$

$$A = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}, C = \begin{bmatrix} 2 & 3 \\ 8 & 6 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1+4 & 5+5 \\ 2+2 & 3+3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 10 \\ 4 & 6 \end{bmatrix}$$

$$(A + B) + C = \begin{bmatrix} 5+2 & 10+3 \\ 4+8 & 6+6 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 13 \\ 12 & 12 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 4+2 & 5+3 \\ 2+8 & 3+6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 8 \\ 10 & 9 \end{bmatrix}$$

$$A + (B + C) = \begin{bmatrix} 1+6 & 5+8 \\ 2+10 & 3+9 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 13 \\ 12 & 12 \end{bmatrix}$$

Thus, $(A + B) + C = A + (B + C)$

3.Existence of Identity in addition of matrices

Let $A = \left[a_{ij}\right]$ is a matrix and *O* be the zero matrix. Let matrices *A,O* has the same order. If *A+O = O+A = A* , then Identity in matrix addition is existed. The matrix *O* is called additive Identity in matrix addition.

4. The existence of additive inverse

Let $A = \left[a_{ij}\right]_{m\times n}$ is a matrix and let $-A = \left[-\left(a_{ij}\right)\right]_{m\times n}$ is a matrix.

If $A + (-A) = (-A) + A = O$ then *-A* is called additive inverse of matrix *A*.

1.1.6.5 Multiplication of Two Matrices

Multiplication of two matrices is possible only when number of rows of the first matrix is equal to the number of columns of the second matrix. Let $A = \left[a_{ij}\right]_{m\times n}$, $B = \left[b_{jk}\right]_{n\times p}$ be two matrices.

Here the number of rows of *A* is equal to the number of columns of *B*. The multiplication of matrices *A,B* is obtained by multiplying term by term entries of the *i*th row of *A* and the *j*th column of *B,* and then summing these products. The entry c_{ik} of the product is obtained for the matrix $C = AB$.

For example, $A = \begin{bmatrix} 3 & 5 \\ 2 & 6 \end{bmatrix}, B = \begin{bmatrix} 7 & -2 \\ 1 & 5 \end{bmatrix},$

$$
\begin{aligned}
C &= AB \\
&= \begin{bmatrix} \rightarrow & \rightarrow \\ 3 & 5 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} \downarrow 7 & -2 \\ \downarrow 1 & 5 \end{bmatrix} \\
&= \begin{bmatrix} 2\times 3 + 5\times 1 & 3\times(-2) + 5\times 5 \\ 2\times 7 + 6\times 1 & 2\times(-2) + 6\times 5 \end{bmatrix} \\
&= \begin{bmatrix} 6+5 & -6+25 \\ 14+6 & -4+30 \end{bmatrix} \\
&= \begin{bmatrix} 11 & 19 \\ 20 & 26 \end{bmatrix}
\end{aligned}
$$

For Example, $A=\begin{bmatrix}6&7&1\\4&2&7\\5&6&8\end{bmatrix}, B=\begin{bmatrix}7&5&2\\1&2&8\\5&5&3\end{bmatrix}$

$$C=AB$$

$$=\begin{bmatrix}6&7&1\\4&2&7\\5&6&8\end{bmatrix}\begin{bmatrix}7&5&2\\1&2&8\\5&5&3\end{bmatrix}$$

$$=\begin{bmatrix}6\times7+7\times1+1\times5 & 6\times5+7\times2+1\times5 & 6\times2+7\times8+1\times3\\4\times7+2\times1+7\times5 & 4\times5+2\times2+7\times5 & 4\times2+2\times8+7\times3\\5\times7+6\times1+8\times5 & 5\times5+6\times2+8\times5 & 5\times2+6\times8+8\times3\end{bmatrix}$$

$$=\begin{bmatrix}42+7+5 & 30+14+5 & 12+56+3\\28+2+35 & 20+4+35 & 8+16+21\\35+6+40 & 25+12+40 & 10+48+24\end{bmatrix}$$

$$=\begin{bmatrix}54&49&71\\65&59&45\\81&77&82\end{bmatrix}$$

1.1.6.6 Properties of Matrix Multiplication

1. Matrix multiplication is associative

Let $A=\left[a_{ij}\right]_{m\times n}, B=\left[b_{ij}\right]_{n\times p}, C=\left[c_{ij}\right]_{p\times r}$ be three matrices, then, $(AB)C=A(BC)$

For example, $A=\begin{bmatrix}2&3\\5&4\end{bmatrix}, B=\begin{bmatrix}1&2\\3&2\end{bmatrix}, C=\begin{bmatrix}-1&2\\1&4\end{bmatrix}$

$$AB=\begin{bmatrix}2&3\\5&4\end{bmatrix}\begin{bmatrix}1&2\\3&2\end{bmatrix}$$

$$=\begin{bmatrix}2+9 & 4+6\\5+12 & 10+8\end{bmatrix}$$

$$=\begin{bmatrix}11&10\\17&18\end{bmatrix}$$

$$(AB)C=\begin{bmatrix}11&10\\17&18\end{bmatrix}\begin{bmatrix}-1&2\\1&4\end{bmatrix}$$

$$=\begin{bmatrix}-11+10 & 22+40\\-17+18 & 34+72\end{bmatrix}$$

$$=\begin{bmatrix}-1&62\\1&106\end{bmatrix}$$

$$BC = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & 2+8 \\ -3+2 & 6+8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 10 \\ -1 & 14 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 1 & 10 \\ -1 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 20+42 \\ 5-4 & 50+56 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 62 \\ 1 & 106 \end{bmatrix}$$

Thus, $(AB)C = A(BC)$

2. Distributive property of Multiplication of matrices over Addition

Let $A = [a_{ij}]_{m \times n}, B = [b_{ij}]_{n \times p}, C = [c_{ij}]_{p \times r}$ be three matrices, then, $A(B+C) = AB + BC$

For example, $A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, C = \begin{bmatrix} -1 & 2 \\ 1 & 4 \end{bmatrix}$

$$B + C = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2+2 \\ 3+1 & 2+4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 4 \\ 4 & 6 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 0+12 & 8+18 \\ 0+16 & 20+24 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 26 \\ 16 & 44 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+9 & 4+6 \\ 5+12 & 10+8 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 10 \\ 17 & 18 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2+3 & 4+12 \\ -5+4 & 10+16 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 16 \\ -1 & 26 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 11 & 10 \\ 17 & 18 \end{bmatrix} + \begin{bmatrix} 1 & 16 \\ -1 & 26 \end{bmatrix}$$

$$= \begin{bmatrix} 11+1 & 10+16 \\ 17-1 & 18+26 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 26 \\ 16 & 44 \end{bmatrix}$$

Thus, $A\left(B + C\right) = AB + BC$

3. Existence of Identity in matrix multiplication

Let $A = \left[a_{ij}\right]_{m \times m}$ is a square matrix. Then there exists an identity matrix I of same order such that $AI = IA = A$

Let $A = \left[a_{ij}\right]_{m \times n}$ be a matrix. There are identity matrices I_n, I_m such that is an $I_m A = AI_n = A$

Self-Assessment Questions

18. Find the $A - B$ for the following matrices:

$$A = \begin{bmatrix} 1 & -2 & 4 \\ 6 & 2 & 5 \\ 2 & 3 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 4 & 1 \\ 3 & -2 & 5 \\ 5 & 6 & 9 \end{bmatrix}$$

(a) $\begin{bmatrix} 1 & -2 & 4 \\ 6 & 2 & 5 \\ 2 & 3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 4 & 1 \\ 3 & -2 & 5 \\ 5 & 6 & 9 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -2 & 4 \\ 3 & 2 & 5 \\ 2 & 3 & -8 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -6 & 3 \\ 3 & 4 & 0 \\ -3 & -3 & -8 \end{bmatrix}$

19. Let $A = \begin{bmatrix} 4 & 2 \\ -1 & 5 \end{bmatrix}, B = \begin{bmatrix} 5 & 2 \\ 1 & -2 \end{bmatrix}$,then find *AB.*

(a) $\begin{bmatrix} 22 & 4 \\ 0 & -12 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 2 \\ -1 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 2 \\ 1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 22 & 4 \\ -1 & 5 \end{bmatrix}$

20. Name the property $A + B = B + A$

(a) Commutative property in matrix multiplication
(b) Existence of Identity in addition of matrices
(c) Commutative property in matrix addition
(d) Associative property in matrix addition.

1.1.7 Transpose of a Matrix

Suppose $A = \left[a_{ij}\right]_{m\times n}$ is a matrix. The transpose of *A* is obtained by interchanging rows and columns. The order of the matrix *A* is $m \times n$ Then the order of the transpose of *A* becomes. The order $n \times m$. The transpose of *A* is denoted by A^T or A'

For example, $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 7 & 6 \end{bmatrix}$

To find the transpose of *A* , it needs to interchange rows and columns. Here *A* is a matrix with order 2x3.the order of the transpose of *A* becomes 3x2.

$$A' = \begin{bmatrix} 1 & 2 \\ 3 & 7 \\ 5 & 6 \end{bmatrix}$$

1.1.7.1 Properties of Transpose of a Matrix

The following are the properties of transpose of a matrix:

1. $(A')' = A$
2. $(kA)' = kA'$
3. $I' = I$,I is a unit vector.
4. $(A + B)' = A' + B'$
5. $(AB)' = B'A'$

1.1.7.2 Symmetric Matrix

Let A is a square matrix. If ,then the matrix A is called symmetric matrix.

For example, $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 1 & 3 & 6 \end{bmatrix}, A' = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 1 & 3 & 6 \end{bmatrix}$.

Here, $A = A'$ so *A* is a symmetric matrix.

1.1.7.3 Skew Symmetric Matrix

Let *A* is a square matrix. If $A = -A'$,then the matrix *A* is called Skew Symmetric matrix.

For example, $A = \begin{bmatrix} 3 & 4 & 7 \\ 4 & 5 & 2 \\ 7 & 2 & 6 \end{bmatrix}, -A' = \begin{bmatrix} 3 & 4 & 7 \\ -4 & 5 & 2 \\ -7 & -2 & 6 \end{bmatrix}$.

Here, $A = -A'$so *A* is a Skew Symmetric matrix.

1.1.7.4 Orthogonal Matrix

Let A is a square matrix. If $AA' = A'A = I$,then the matrix A is called Orthogonal matrix.

For example, $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, A' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

$$AA' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A'A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Here, $AA' = A'A = I$ so *A* is a Orthogonal matrix.

Self-Assessment Questions

21. Find the transpose of the following matrix.

$$A = \begin{bmatrix} 2 & -1 & 4 \\ 3 & 4 & 1 \\ 5 & 3 & 6 \end{bmatrix}$$

(a) $\begin{bmatrix} 2 & -1 & 4 \\ 3 & 4 & 1 \\ 5 & 3 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 3 & 4 \\ -1 & 4 & 1 \\ 5 & 3 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 1 & 6 \\ 3 & 2 & 5 \\ 2 & 3 & -8 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 3 & 5 \\ -1 & 4 & 3 \\ 4 & 1 & 6 \end{bmatrix}$

22 Give one example of skew symmetric matrix.

(a) $\begin{bmatrix} 0 & 2 & 3 \\ 2 & 0 & 2 \\ 3 & 2 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 3 & 4 \\ -1 & 4 & 1 \\ 5 & 3 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 1 & 6 \\ 3 & 2 & 5 \\ 2 & 3 & -8 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 3 & 5 \\ -1 & 4 & 3 \\ 4 & 1 & 6 \end{bmatrix}$

23. Give one example of orthogonal matrix.

(a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$

Summary

- In this **Unit**, we defined matrix as a rectangular arrangement of numbers or functions in rows and columns.
- We discussed the types of matrices like Column matrix, Row matrix, Rectangular matrix, Square matrix, Diagonal matrix, Scalar matrix, Identity matrix, Zero matrix, Triangular matrices, Symmetric matrix, Skew symmetric matrix, and Orthogonal matrix.
- We learnt scalar multiplication of a matrix which obtained by multiplying each element of a matrix by a scalar.
- We discussed equality of matrices. Two matrices A, B with same order is said to be equal matrices if each element of A is equal to the corresponding element of B.
- We learnt matrix operations of addition of matrices, subtraction of matrices and multiplication of matrices.
- We discussed transpose of a matrix which means obtaining a new matrix by interchanging rows and columns of the original matrix.
- We learnt the properties of transpose of a matrix.

Key words

Column matrix
Row matrix
Rectangular matrix
Square matrix
Diagonal matrix
Scalar matrix
Identity matrix
Zero matrix or null matrix
Triangular matrices
Transpose of a matrix
Symmetric matrix
Skew symmetric matrix
Orthogonal matrix

Terminal Questions

1. Find the order of the matrices.

(i) $\begin{bmatrix} 4 & 3 \\ 2 & -1 \\ 5 & 6 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 & 4 \\ 7 & 6 & -2 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & 6 \\ 4 & -2 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 3 & 5 \\ 6 & 7 & 8 \\ 4 & 5 & 2 \\ 3 & 2 & -1 \end{bmatrix}$

2. Find the Lower and upper triangular matrices for the following.

(i) $\begin{bmatrix} 2 & 3 & 5 & 1 \\ -1 & 2 & 4 & 3 \\ 5 & 6 & 1 & 2 \\ -2 & 3 & 4 & 5 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 5 & 2 \\ 3 & 4 & 6 \\ -2 & 4 & 3 \end{bmatrix}$

3. Let $A=\begin{bmatrix}3 & 4 & 5\\ -6 & 7 & 8\\ 9 & 3 & 2\end{bmatrix}$ Find $5A,-2A$

4. Let $A=\begin{bmatrix}2 & a & b\\ c & 3 & 5\\ 2 & 4 & 3\end{bmatrix}$, $B=\begin{bmatrix}2 & -2 & 1\\ 5 & 3 & 5\\ 2 & 4 & 3\end{bmatrix}$ be two equal matrices. Then find a,b,c

5. Suppose $A=\begin{bmatrix}4 & 1 & 3\\ 5 & 2 & 5\\ 2 & 3 & 3\end{bmatrix}, B=\begin{bmatrix}2x & x-1 & 3\\ 5 & x & 5\\ 2 & 3 & x+1\end{bmatrix}$ and A, B are equal matrices.

Then find the value of x.

6. Suppose $M=\begin{bmatrix}1 & 2 & 3\\ 4 & 3 & 5\\ 2 & 9 & 4\end{bmatrix}, N=\begin{bmatrix}5 & 1 & 7\\ 8 & 9 & 2\\ -2 & 1 & 1\end{bmatrix}$. Show that matrix addition is commutative.

7. Suppose $P=\begin{bmatrix}2 & -1 & 4\\ 6 & 4 & 5\\ 3 & 5 & -2\end{bmatrix}, Q=\begin{bmatrix}3 & 2 & 8\\ 5 & 4 & 3\\ -3 & 2 & 1\end{bmatrix}$, find *PQ*.

8. Suppose $A=\begin{bmatrix}2 & 3\\ 1 & 4\end{bmatrix}, B=\begin{bmatrix}4 & 5\\ -2 & 6\end{bmatrix}, C=\begin{bmatrix}2 & 3\\ 8 & 7\end{bmatrix}$. Show that matrix multiplication is associative.

9. Let $L=\begin{bmatrix}1 & 2\\ 1 & 3\end{bmatrix}, M=\begin{bmatrix}-1 & 4\\ 5 & 3\end{bmatrix}, N=\begin{bmatrix}3 & -2\\ 6 & 7\end{bmatrix}$ be the matrices. Show that distributive property of multiplication of matrices over addition holds:

10. Let $A=\begin{bmatrix}2 & 3 & 1\\ 5 & 4 & 3\\ -2 & 3 & 2\end{bmatrix}$. Prove that $(A')'=A$

11. Suppose $A=\begin{bmatrix}1 & -2\\ 5 & 4\end{bmatrix}, B=\begin{bmatrix}6 & 4\\ 1 & -3\end{bmatrix}$ Show that $(AB)'=B'A'$

12. Give one example of symmetric matrix and one example of skew symmetric matrix.

Answer Keys

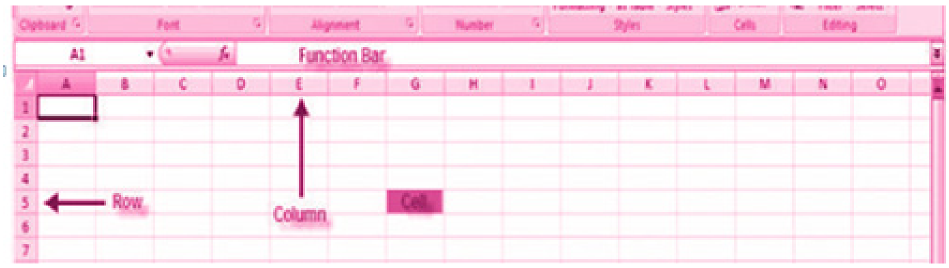
Self-assessment Questions	
Question Number	Answer
1	d
2	b
3	c
4	a
5	b
6	d
7	c
8	a
9	b
10	a
11	c
12	b
13	d
14	a
15	d
16	b
17	b
18	d
19	a
20	c
21	d
22	d
23	b

Activities

Activity Type: Offline **Time: 40 Minutes**

1. Collect 5 people salaries, expenses and their savings in a month to nearest thousands of rupees in an electronic spread sheet. Write the data on a paper this makes a 5×3 matrix.

2. Collect the data of 10 progress reports of students in a school from the class X. Find the average pass percent of each subject. The minimum pass mark of each subject is 40 percent.



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External Resources

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Alpha. C(2009). *Mathematics for Economists*(10th ed.). Tata McGraw-Hill.

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Video Links

Topic	Link
Matrix definition	https://www.youtube.com/watch?v=1R5opPSokck
Types of matrix	https://www.youtube.com/watch?v=QqmO2vN2KZ4
Matrix addition and subtraction	https://www.youtube.com/watch?v=QXUbFzEd3Ww
Multiplication of matrices	https://www.youtube.com/watch?v=XaChSDBF_7s

BUSINESS MATHEMATICS

https://www.kluniversity.in/cdoe/
supportcdoe@kluniversity.in